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HIGH-FREQUENCY ASYMPTOTICS OF ACOUSTIC PRESSURE FOR BOUNDED WAVE BEAM SCATTERING BY AN ELASTIC SPHERE*

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Asymptotic high-freqeuncy estimates are obtained for the amplitudes of specular and non-specular reflections with extraction of the contribution of sound reradiation into the surrounding medium by Rayleigh type surface elastic waves. The conditions are found that govern the magnification of scattering in the opposite direction. The theoretical explanation of the book reflection effect /1/ for bounded sound beam incidence on the plane interface of a fluid-elastic solid is given by many authors in different situations (/12/, say). As for non-specular reflection of a plane sound wave by bounded elastic bodies (plates, cylinders, rods, and shells enclosed in a screen), studied most thoroughly in /3-9/, this effect is a consequence of satisfying the space-time resonance conditions between the incident acoustic wave and the normal surface waves excited in an elastic solid under total internal reflection.

It is interesting to clarify and describe the book reflection of a bounded sound beam incident on the curvilinear interface between two media. Selection of the contributions of surface waves in the echo signal from elastic cylinders was carried out experimentally /10, 11/ by sounding a narrow part of an object surface by a pencil beam near the critical angles of surface acoustic wave excitation. An analytic description of such a process was given in /12/ for analogous wave excitation conditions in the case of spherical and cylindrical elastic reflectors. However, the echo signals reradiated by the surface waves were only examined in the domain of the geometric shadow of the objects. Non-specular reflection in the reverse direction directly from the sounded section of the interfacial boundary without preliminary residency in the shadow domain was not analysed.

1. Let a sound beam, whose effective transverse section near the interface of two media is represented as a narrow circular ring of width ν_i, impinge on an elastic object of spherical shape that is in an ideal compressible fluid. The acoustic pressure of the incident beam is expressed by the formula

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$$p_{inc}(r, \theta, \omega) = e^{ikr\cos\theta} H(\xi_+ - \xi) H(\xi - \xi_-) \tag{1.1}$$

$$\xi = r\sin\theta, \quad \xi_\pm = \xi_i \pm {}^1/_2 \nu_i$$

where the normalization constant is taken to be equal to one, the time factor $e^{-i\omega t}$ is omitted, ξ_i is the impact parameter of the central rays of the beam, r, θ are spherical coordinates with origin at the centre of the object, $k = \omega/c$ is the wave number, ω is the vibration frequency, c is the velocity of sound in the fluid, and $H(x)$ is the Heaviside function. The impinging wave sounds the surface of a sphere $r = a$ along a ring $0 \leqslant \theta_- \leqslant \pi - \theta \leqslant \theta_+, \theta_\pm = \theta_i \pm \theta_0/2$, where θ_i is the beam sighting angle, and θ_0 is the angular width of this ring. We will find the complex amplitude of the acoustic pressure in the scattered wave in the form of an expansion in partial waves /12/

$$p_{sc}(r, \theta, \omega) = i \sum_{l=0}^{\infty} \left(l + \frac{1}{2}\right) f_l^\circ(x) h_l^{(1)}(kr) P_l(\cos\theta), \quad x = ka \tag{1.2}$$

$$f_l^\circ(x) = i \frac{j_l^\circ(x) F_l(x) - x j_l^{\circ\prime}(x)}{h_l^{(1)}(x) F_l(x) - x h_l^{(1)\prime}(x)} \tag{1.3}$$

$$j_l^\circ(x) = \int_{\pi-\theta_+}^{\pi-\theta_-} e^{ix\cos\alpha} P_l(\cos\alpha) \sin\alpha \, d\alpha \tag{1.4}$$

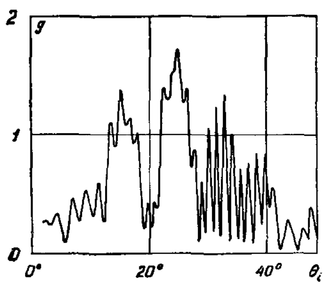

Fig.1

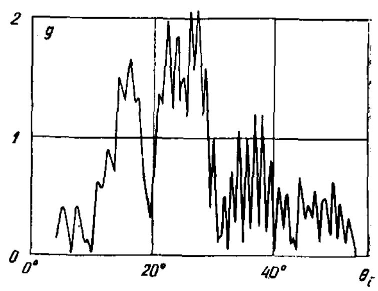

Fig.2

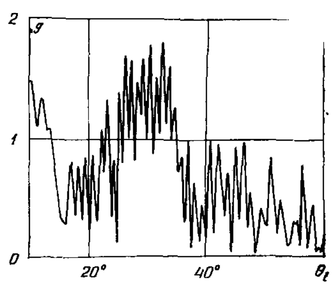

Fig.3

Here $h_l^{(1)}(x)$ is the spherical Hankel function of the first kind, $P_l(\cos\theta)$ is the Legendre polynomial, $f_l^\circ(x)$ is the partial scattering amplitude, and $F_l(x)$ is the compliance function of the obstacle, that depends on the wave dimensions and the physico-mechanical parameters of the object /13/. The sound pressure in the wave field $(r \to \infty)$ is characterized by the scattering amplitude $f(\theta, k)$:

$$p_{sc}(r, \theta, \omega) = r^{-1} e^{ikr} f(\theta, k) \tag{1.5}$$

$$f(\theta, k) = \frac{1}{k} \sum_{l=0}^{\infty} (-i)^l \left(l + \frac{1}{2}\right) f_l^\circ(x) P_l(\cos\theta)$$

Let us examine the dependence of the moduli of the scattering amplitudes on the beam sighting angle θ_i for a steel sphere submerged in water /13/ with the wave radius $x = 314.36$ for values of the beam sweep angle θ_0 equal to 4, 8, and 20° (Figs.1, 2 and 3, respectively). The scattering amplitudes are calculated in the location direction with extraction of the potential scattering corresponding to the case of an acoustically rigid fixed sphere $g(\pi, k) =$

$|f(\pi, k) - f^{(r)}(\pi, k)| / (2a)$. The beams are well collimated, since 3.5 incident wave lengths are packed along the width of the sounded ring on the sphere surface even for $\theta_0 = 4°$. Scanning by the beam sighting angle from small to large values for a fixed beam width successively clarifies the domain of magnified reverse reflection of whispering-gallery waves of the longitudinal type $(\theta_i \approx 15°)$ and the transverse type $(\theta_i \approx 26°)$, as well as of Rayleigh waves $(\theta_i \approx 27.4°)$. The wave dimensions selected for the sphere correspond approximately to disposition of the Rayleigh spectrum line of one of the partial scattering amplitudes $(l = 142)$. Hence the scattering amplitude fluctuations in the domain of the critical excitation angle of Rayleigh-type surface waves are resonance in nature. They are due to the passage of the sound spot through the Fresnel quarter-wave zone during scanning at the sighting angle, and to the absence of the contribution of other kinds of waves. Scattering amplitudes are considerably influenced by refracted waves of longitudinal, transverse, and mixed, longitudinal-transverse types rereflected a multiple number of times from within by the sphere surface in the domain of critical whispering-gallery wave excitation angles.

We note that back-scattering for plane-wave incidence on a sphere occurs because of multiple diffraction of perispherical waves and rereflected bulk waves. Pencil-beam sounding of a deformable solid by a bounded beam discloses magnified non-specular reflection directly from the excited part of the surface when the space-time synchronization conditions between the incident beam and the reradiated waves are satisfied.

2. It is known that the formation of a pencil-beam sound within linear acoustic limits is possible only for sufficiently high frequencies of the time spectrum $x \gg 1$. The angular spectrum band should obviously also be high-frequency, i.e., partial waves with large values of the angular moments l should be excited. Under these conditions, an asymptotic analysis of the solution of the scattering problem is possible.

We will apply Poisson's formula /14/ to the series (1.2). We then obtain

$$p_{sc}(r, \theta, \omega) = \sum_{m=-\infty}^{\infty} (-1)^m p_{sm}(r, \theta, \omega), \quad p_{sm}(r, \theta, \omega) = \tag{2.1}$$
$$i\int_0^\infty \lambda X(\lambda)\, h^{(1)}_{\lambda-1/2}(kr)\, P_{\lambda-1/2}(\cos\theta)\, e^{2im\lambda}\, d\lambda \quad (m = 0, \pm 1, \pm 2, \ldots)$$

where $X(\lambda)$ is an interpolating function /14/ that goes over for $\lambda = l + 1/2$ into $f_l^\circ(x)$, $P_{\lambda-1/2}(z)$ is the Legendre function of the first kind.

It can be shown by asymptotic evaluation of the integrals that the series (2.1) describes a multiple scattering process (with multiplicity $|m| \gg 1$) caused by sound-wave diffraction by the closed surface of an object, and rereflection of elastic waves for sound transmission within the obstacle.

We will examine the zero-th term in these formulas taking boundedness of the incident beam into account. Using relationships (1.5), we find in the wave zone

$$p_{s0}(r, \theta, \omega) = r^{-1} e^{ikr} f_0(\theta, k) \quad (r \to \infty) \tag{2.2}$$
$$f_0(\theta, k) = \frac{1}{k} e^{\pi i/4} \int_0^\infty \lambda X(\lambda)\, e^{-\pi i\lambda/2} P_{\lambda-1/2}(\cos\theta)\, d\lambda$$

We introduce the dimensionless impact parameter $\xi = \lambda/x$ $(\lambda \gg 1,\ x \gg 1;\ 0 < \xi < 1)$. Evaluating the incomplete Bessel spherical functions $j^\circ_{\lambda-1/2}(x)$ in the functions $X(\lambda)$ by the saddle-point method (see Formula (1.4) for $l = \lambda - 1/2$), we find

$$f_0(\theta, k) = f_{01}(\theta, k) + f_{02}(\theta, k) \tag{2.3}$$
$$f_{01}(\theta, k) = \frac{x^2}{k} \int_{\sin\theta_-}^{\sin\theta_+} \exp\left[-2ix\left(\sqrt{1-\xi^2} - \xi \arccos\xi\right)\right] \tau(\xi)\, P_{x\xi-1/2}(\cos\theta)\, \xi\, d\xi$$
$$f_{02}(\theta, k) = \frac{x}{k} \sqrt{\frac{2}{\pi}\sin\theta_i}\, \exp(-ix\cos\theta_i) \sum_{\nu=\pm 1} \exp[-\pi i(\nu+1)/4] \times$$
$$\int_0^\infty \sqrt{\xi}\, \sqrt[4]{1-\xi^2}\, \exp\langle -ix\{\sqrt{1-\xi^2} + \xi[\arcsin\xi - \nu(\pi - \theta_i)]\}\rangle \times$$
$$\tau(\xi)\, \Phi_\nu(\xi)\, \Psi(\xi)\, P_{x\xi-1/2}(\cos\theta)\, \xi\, d\xi$$
$$\tau(\xi) = -\frac{F_{x\xi-1/2}(x) + ix\sqrt{1-\xi^2}}{F_{x\xi-1/2}(x) - ix\sqrt{1-\xi^2}}, \quad \Phi_\nu(\xi) = \frac{\sin[1/2\, x\theta_0(\sin\theta_i - \nu\xi)]}{\sin\theta_i - \nu\xi}$$

$$\Psi(\xi) = \frac{F_{x\xi^{-1/_3}}(x) + ix\cos\theta_i}{F_{x\xi^{-1/_3}}(x) + ix\sqrt{1-\xi^2}}$$

where $\tau(\xi)$ describes the high-frequency reflected, rereflected, and surface acoustic waves. Thus, for instance, we have for the case of a continuous elastic sphere

$$\tau(\xi) = R_{12}(\xi) + f_2(\xi)/[1 - f_1(\xi)] \tag{2.4}$$

where R_{12} is the reflection coefficient of the sound wave reflected from the interface of the media, and the functions f_1, f_2 /16/ correspond to the contributions of waves rereflected within the scatterer.

We assume that sounding of the sphere occurs along a part of its surface near the critical angle of Rayleigh-wave excitation. Then relationships (2.4) become

$$\tau(\xi) \approx R_{12}(\xi) \approx -R^*(\xi)/R(\xi), \quad R(\xi) = B(\xi) + N_s A(\xi) \tag{2.5}$$

$$A(\xi) = i\frac{c\sqrt{\xi_L^2-1}}{c_L\sqrt{1-\xi^2}} + \frac{1}{x\sqrt{1-\xi^2}}\left[\frac{c(2-\xi^2)\sqrt{\xi_L^2-1}}{c_L\sqrt{(1-\xi^2)^3}} -\right.$$

$$\left. i\left(2\zeta - 2\xi_T^2 + \frac{\xi_L^2-2}{2(\xi_L^2-1)}\right)\right] + O(x^{-2}), \quad B(\xi) = (1-2\xi_T^2)^2 - 4\xi_T^2\zeta - \frac{2c_T}{x_T c} \times$$

$$\left[\left(1 + \xi_T^2 - \frac{2\xi_T^2}{\xi_L^2-1}\right)\sqrt{\xi_T^2-1} + \frac{c_T}{c_L}\left(\xi_T^2 + \frac{2}{\xi_T^2-1}\right)\sqrt{\xi_L^2-1}\right] + O(x^{-2})$$

$$N_s = \frac{\rho}{\rho_s}, \quad \xi_A = \frac{c_A}{c}\xi > 1 \quad (A = L, T), \quad \zeta = \frac{c_T}{c_l}\sqrt{\xi_L^2-1}\sqrt{\xi_T^2-1}$$

where ρ and ρ_s are the density of the surrounding fluid (gas) and the material of the sphere, c_L and c_T are the longitudinal and transverse wave propagation velocities in the elastic sphere, and the asterisk denotes the complex conjugate quantity.

The pole of the function $\tau(\xi)$ (A Regge pole of Rayleigh type) is determined by solving the dispersion equation $R(\xi) = 0$, where for large x this solution can be obtained by successive approximations /17/

$$\xi = \xi_R = \frac{c}{c_T}[\eta_T + \beta x^{-1} + O(x^{-2})] = \sin\bar\theta_R, \quad \bar\theta_R = \theta_R + i\nu_R \tag{2.6}$$

$$\beta = -\frac{1}{8\eta_T}\left\{2\frac{c_T}{c}\left[\frac{c_T}{c_L}\left(\eta_T^2 + \frac{2}{\mu_T^2}\right)\mu_L - \left(1 + \eta_T^2 - \frac{2\eta_T^2}{\mu_L^2}\right)\mu_T\right] + \right.$$

$$\left.\frac{N_s}{\mu}\left[\frac{c\mu_L(2-\eta^2)}{2c_L\mu^3} - 2i\left(\frac{c_T}{c_L}\mu_L\mu_T - \eta_T^2 - \frac{\eta_L^2-2}{4\mu_L^2}\right)\right]\right\} \times$$

$$\left[1 - 2\eta_T^2 + \frac{c_T}{c_L}\mu_L\mu_T + \frac{1}{2}\eta_T^2\left(\frac{c_L\mu_T}{c_T\mu_L} + \frac{c_T\mu_L}{c_L\mu_T}\right) - \frac{iN_s c^2}{8c_L c_T\mu}\left(\frac{c_L^2}{c_T^2\mu_L} + \frac{\mu_T}{\mu^2}\right)\right]^{-1}$$

$$\eta_A = \frac{c_A}{c}\eta, \quad \mu_A = \sqrt{\eta_A^2-1} \quad (A = L, T), \quad \mu = \sqrt{1-\eta^2}$$

where $\eta = \xi_R^\infty$ is the solution of the dispersion equation for the case of a plane interface for an acoustic-fluid-elastic half-space $\xi_R^\infty = \bar c/\bar c_R$, $\bar c_R$ is the complex Rayleigh-wave velocity /18/, and $\bar\theta_R$ is the complex critical angle of surface elastic wave excitation on a sphere in a fluid. In particular, we find for $N_s \ll 1$

$$\eta_T = \varepsilon_T + \alpha_T, \quad \varepsilon_T = \frac{c_T}{c_R^\infty} \approx \frac{1+\nu_s}{0{,}862 + 1{,}14\nu_s}, \quad \varepsilon_L = \frac{c_L}{c_R^\infty} \tag{2.7}$$

$$\alpha_T = -\frac{iN_s c_L\gamma_L}{8c\varepsilon_T\gamma}\left[1 - 2\varepsilon_T^2 + \frac{c_T}{c}\gamma_L\gamma_T + \frac{1}{2}\varepsilon_T^2\left(\frac{c_L\gamma_T}{c_T\gamma_L} + \frac{c_T\gamma_L}{c_L\gamma_T}\right)\right]^{-1}$$

$$\gamma = \sqrt{1-\varepsilon^2}, \quad \varepsilon = \frac{c}{c_R^\infty}, \quad \gamma_A = \sqrt{\varepsilon_A^2-1} \quad (A = L, T)$$

Here c_R^∞ is the Rayleigh-wave velocity for an elastic half-space adjoining a vacuum /18/ and ν_s is Poisson's ratio of the material.

On the basis of (2.5)-(2.7), we represent the reflection coefficient near the Regge pole in the form

$$\tau(\xi) \approx R_{12}(\xi) \approx -\frac{R^{*\prime}(\xi_R^*)}{R'(\xi_R)}\frac{\xi - \xi_R^*}{\xi - \xi_R} \tag{2.8}$$

where ξ_R^* is the zero of the function $R^*(\xi)$, complex conjugate to ξ_R.

We also perform an asymptotic evaluation of the integrals in the functions f_{01} and f_{02} from (2.3) by the saddle-point method. If the Regge pole is sufficiently remote from the real axis of the complex plane of the variable $\zeta = \xi + i\,\mathrm{Im}\,\zeta$, then the principal part of the asymptotic is determined by the contribution from the saddle point ξ_s. For $\mathrm{Im}\,\xi_R \ll 1\ (\nu_R \ll 1)$ the case should also be considered when the Regge pole is near the saddle point whose location on the real axis depends on the scattering angle θ /19/. Thus we obtain

$$f_{01}(\theta, k) \approx \frac{1}{2} a \exp\left[-2ix\cos\left(\frac{\pi-\theta}{2}\right)\right] R_{12}(\xi_s) H\left(\theta_0 - \left|\frac{\pi-\theta}{2} - \theta_i\right|\right) \tag{2.9}$$

for the function f_{01} for $\mathrm{Im}\,\xi_R$ not small and taking into account that $\xi_s = \sin[(\pi-\theta)/2]$ i.e., the value θ satisfies the approximate equality $\pi - \theta \approx 2\theta_i$ $(\theta_0 \ll \theta_i)$. Formula (2.9) therefore corresponds to specular reflection. According to (2.6) and (2.8) the maximum magnification of a specularly reflected wave occurs when the beam is incident at the critical Rayleigh angle: $\theta_i \approx \theta_R$. We arrive at an analogous conclusion also in the case when $\nu_R \ll 1$.

We will now examine the function f_{02}. For $x \gg 1$, $\lambda\theta \gg 1$ we find

$$f_{02}(\theta, k) \approx \frac{1}{\pi k}\sqrt{\frac{\sin\theta_i}{ix\sin\theta}} \exp(-ix\cos\theta_i) \sum_{\nu,\chi=\pm1} \exp\left[-\frac{i\pi}{4}(\nu+\chi)\right] \times \tag{2.10}$$
$$\int_0^\infty f_\nu(\xi)\exp[-ixq_{\nu\chi}(\xi)]\,d\xi$$
$$f_\nu(\xi) = \xi\sqrt{1-\xi^2}\,\Phi_\nu(\xi)\Psi(\xi)R_{12}(\xi)$$
$$q_{\nu\chi}(\xi) = \sqrt{1-\xi^2} + \xi\,[\arcsin\xi - \nu(\pi-\theta_i) - \chi\theta]$$

Since only two positive roots $\xi_\nu = \nu\sin(\pi-\theta-\theta_i)$, are determined from the equation $q_{\nu\chi}'(\xi) = 0$ where $\pi - \theta \geqslant \theta_i$ if $\nu = 1, \chi = -1$, and $0 \leqslant \pi - \theta \leqslant \theta_i$ if $\nu = -1, \chi = 1$, then to obtain the principal terms of the asymptotic expansions in (2.10) it is sufficient to confine oneself to examining the terms for $\nu = 1, \chi = -1$ and $\nu = -1, \chi = 1$. Evaluating the integrals occurring here by the saddle-point method under the conditions that the Regge pole can be near the saddle point /19/, we obtain

$$f_{02}(\theta, k) \approx \sum_{\nu=\pm1} F_\nu(\theta, k) \tag{2.11}$$
$$F_\nu(\theta, k) = \exp\{ix[\cos(\pi-\theta_i) + \cos(\pi-\theta-\theta_i)]\} \times \tag{2.12}$$
$$\{a_\nu\,\mathrm{sgn}(\mathrm{Im}\,b_\nu)\exp(-xb_\nu^2)\,\mathrm{erfc}\,[-ib_\nu\sqrt{x}\,\mathrm{sgn}(\mathrm{Im}\,b_\nu)] +$$
$$(\pi x)^{-1/2}T_\nu\}\,H[\nu(\pi-\theta-\theta_i)]$$
$$a_\nu = \frac{\sin\theta_R}{2k}\sqrt{\frac{\sin\theta_i}{ix\sin\theta\cos\theta_R}}(\cos\theta_R + \cos\theta_i)\Phi_\nu(\xi_R)\operatorname*{res}_{(\xi=\xi_R)} R_{12}(\xi)$$
$$b_\nu = e^{-\pi i/4}\{\cos(\pi-\theta-\theta_i) - \cos\bar\theta_R - [\bar\theta_R - \nu(\pi-\theta-\theta_i)]\sin\bar\theta_R\}^{1/2}$$
$$T_\nu = hf_\nu(\xi_\nu) + \frac{a_\nu}{b_\nu}, \qquad h = e^{-\pi i/4}\sqrt{2\cos(\pi-\theta-\theta_i)}$$

As $|b_\nu|\sqrt{x} \to \infty$ $(\nu = \pm1)$, when the Regge pole (2.5) is remote from the real axis $(\nu_R \gg 1)$, we find from (2.12)

$$F_\nu(\theta, k) \approx \frac{1}{kx}\sqrt{\frac{2\sin\theta_i}{\sin\theta\cos(\pi-\theta-\theta_i)}}\exp(-ix\cos\theta_i) \times \tag{2.13}$$
$$f_\nu(\xi_\nu)H[\nu(\pi-\theta-\theta_i)] = O(x^{-2}), \quad \nu = \pm1$$

i.e., the contribution to the scattered field (2.3) is formed because of the specular reflection (2.9).

If $|b_\nu|\sqrt{x}$ is not a large quantity, then the behaviour of the functions F_ν is determined mainly because of the exponential function and the function in (2.12). Thus, for $|b_\nu|\sqrt{x} \to 0$ $(\nu = \pm1)$

$$F_\nu(\theta, k) \approx \exp(-ix\cos\theta_i)\,[a_\nu\,\mathrm{sgn}(\mathrm{Im}\,b_\nu)\exp(-xb_\nu^2) + \tag{2.14}$$
$$\sqrt{\pi/x}\,T_\nu]\,H[\nu(\pi-\theta-\theta_i)]$$

If the Regge pole (2.6) here has a small imaginary part, the conditions

$$b_\nu^2 \approx \alpha_\nu - i\beta_\nu + O(\nu_R^4) \to 0 \quad (\nu = \pm 1)$$
$$\alpha_\nu = \nu_R \{(1 + {}^1/_6\nu_R^2)[\nu(\pi - \theta - \theta_i) - \theta_R]\cos\theta_R - {}^1/_3\nu_R^2 \sin\theta_R\}$$
$$\beta_\nu = \cos(\pi - \theta - \theta_i) - (1 - {}^1/_2\nu_R^2)\cos\theta_R + (1 + {}^1/_2\nu_R^2) \times$$
$$[\nu(\pi - \theta - \theta_i) - \theta_R]\sin\theta_R$$

are satisfied if $\pi - \theta \approx \theta_i + \theta_R$ for $\nu = 1$ and $\pi - \theta \approx \theta_i - \theta_R > 0$ for $\nu = -1$.

Analysis of expressions (2.12) and (2.14) shows that an exponential increase in the moduli of the complex amplitudes occurs if

$$\pi - \theta < \theta_i + \theta_R + \Delta\theta \ (\nu = 1),\ \pi - \theta > \theta_i - \theta_R - \Delta\theta \geqslant 0 \ (\nu = -1) \tag{2.15}$$
$$\Delta\theta = 2\nu_R^2 \operatorname{tg}\theta_R/(6 + \nu_R^2)$$

where it is necessary that the sighting angle of the incident beam θ_i exceeds somewhat the critical angle of the surface wave θ_R to satisfy the condition at $\nu = -1$. We note that the effect of amplitude growth is negligible since the quantity $|b_\nu|\sqrt{x}$ grows rapidly when (2.15) are satisfied and (2.14) loses its meaning, i.e., the asymptotic form (2.13) becomes valid. For a fixed x the magnification effect is determined by the function T_ν. The values of $|T_\nu|$ are important if

$$\pi - \theta = \theta_i + \theta_R + \Delta\theta \ (\nu = 1),\ \pi - \theta = \theta_i - \theta_R - \Delta\theta \geqslant 0 \ (\nu = -1) \tag{2.16}$$

Here

$$b_\nu \approx e^{-\pi i/4}\nu_R \sqrt{{}^1/_2\cos\theta_R} = O(\nu_R) \quad (\nu = \pm 1)$$

It is seen from (2.12) that the greatest magnification of the function T is obtained if $\theta_i = \theta_R + \Delta\theta$, since then

$$a_1 \approx [\operatorname{sh}({}^1/_2 x\theta_0\nu_R\cos\theta_R)/(\nu_R\sqrt{\cos\theta_R})] \operatorname{res} R_{12}(\sin\bar\theta_R)$$

Therefore, an exponential magnification of the function F_1, and therefore, f_{02} also, occurs as a result of Rayleigh-wave reradiation for the scattering angle $\pi - \theta = 2\theta_i + \Delta\theta$, i.e., in the specularly reflected wave direction. At the same time the function a_{-1} is not magnifying for $\theta_i \approx \theta_R$. However, it follows from conditions (2.16) that the equality $\theta_i = \theta_R + \Delta\theta$ corresponds to the scattering angle $\theta = \pi$, i.e., reverse (non-specular) reflection occurs here. A significant increase in $|f_{02}|$ occurs near $\theta = \pi - \delta$ $(0 < \delta \ll 1)$ because of the focusing factor contained in the function a_{-1} in (2.12).

We note that the asymptotic formulas (2.12) are not satisfied in this situation. Consequently, the behaviour of the scattering amplitude $f_{02}(\theta, k)$ is examined separately for $\theta \to \pi$.

3. If the acoustic signals are taken for angles $\theta \approx \pi$, then the expression for $f(\theta, k)$ from (1.5) can be conveniently rewritten in the form

$$f(\theta, k) = \frac{1}{k}\sum_{l=0}^{\infty}\left(l + \frac{1}{2}\right)e^{i\pi l/2} f_l^\circ(x) P_l(-\cos\theta) \tag{3.1}$$

Furthermore, using Poisson's formula and satisfying (2.1)-(2.3) by analogy with the relationships (1.5), we obtain the asymptotic evaluation of the integrals ($J_0(z)$ is the Bessel function)

$$f(\theta, k) \approx \pi i a_0 \operatorname{sgn}(\operatorname{Im} b)\exp(-xb_\nu^2)\operatorname{erfc}[-ib\sqrt{x}\operatorname{sgn}(\operatorname{Im} b)] + \sqrt{\pi/x}\, T_0 \tag{3.2}$$
$$b = e^{-\pi i/4}\sqrt{\cos\theta_i - \cos\bar\theta_R - (\bar\theta_R - \theta_i)\sin\bar\theta_R}$$
$$a_0 = \sqrt{\frac{2x(\pi - \theta)\sin\theta_i \operatorname{tg}\theta_R}{\sin\theta}}\, \frac{\nu_R \sin\theta_R}{k}\, \frac{\cos\theta_R + \cos\theta_i}{\sin\theta_R + \sin\theta_i} \times$$
$$\sin\left[\frac{1}{2}x\theta_0(\sin\bar\theta_R + \sin\theta_i)\right](R^{*\prime}(\xi_R^*)/R'(\xi_R))\, J_0[x(\pi - \theta)\sin\bar\theta_R]$$
$$T_0 = \exp^{\pi i/4}\sqrt{2\cos\theta_i}\, f(\sin\theta_i) + a_0/b$$
$$f(\sin\theta_i) = -\frac{1}{2k}\sqrt{\frac{x(\pi - \theta)\sin\theta_i \sin 2\theta_i}{\sin\theta}}\sin(x\theta_0\sin\theta_i) \times$$
$$R_{12}(\sin\theta_i)\, J_0[x(\pi - \theta)\sin\theta_i]\exp(-\pi i\cos\theta_i)$$

Analysis of (3.2) shows that for $\varepsilon_0 = |\theta_i - \theta_R| \neq 0$ the modulus of the scattering amplitude for $\nu_R > 0$ has the form: $|f(\theta, k)| \sim \exp(-x\nu_R\varepsilon_0\cos\theta_R)$. The magnifying effect of non-specular reflection is obtained for $\theta_i \approx \theta_R$, $\nu_R \ll 1$; then (1.3) becomes

$$f(\theta, k) \approx \frac{\pi i \nu_R}{k} \sqrt{\frac{2x(\pi-\theta)\cos\theta_R}{\sin\theta}} \sin\theta_R \sin(x\theta_0 \sin\theta_R)\,(R^{*\prime}(\xi_R^*)/R'(\xi_R)) \times \tag{3.3}$$
$$J_0[(\pi-\theta)x\sin\theta_R]\exp[-ix(1+{}^1/_2\nu_R^2)\cos\theta_R] \times$$
$$\Big\{\operatorname{erfc}(u) + \frac{1}{\pi}\exp[(-u^2)\sum_{n=-\infty}^{\infty}{}' \frac{\exp(-n^2/4)}{n^2+4u^2}[-2u+\exp(-2iu^2)\times$$
$$(2u\operatorname{ch} nu + in\operatorname{sh} nu)]\Big\},\quad u = {}^1/_2\nu_R\sqrt{x\cos\theta_R}$$

It follows from (3.3) that the maximum value of the modulus of the non-specular reflection amplitude, as well as of the location scattering cross-section, will hold when the number of waves, a multiple of a quarter of a Rayleigh surface wavelength, is stacked along the width of the ring being sounded on the sphere surface

$$a\theta_0 \approx {}^1/_2\,(n + {}^1/_2)\,\lambda_R\ \ (n = 0, 1, 2, \ldots),\ \lambda_R = 2\pi a/\sin\theta_R$$

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